

## MEDIUM 1

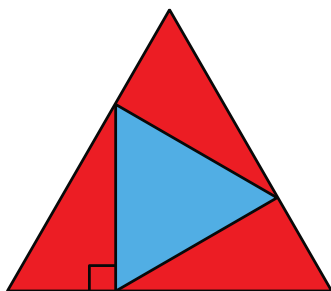
$$(\blacksquare \blacktriangle + 1) / 2 = \blacktriangle \blacksquare$$

Isaac takes a 2-digit number, adds 1 to it and divides by 2. The result is Isaac's number but with the digits reversed. What was Isaac's original number?

Answer: 73

Solution: If the digits of Isaac's number are  $AB$  then the final number is  $BA$  and so we must have  $(10A + B + 1)/2 = 10B + A$ . Multiplying out this gives  $8A + 1 = 19B$ . Since  $8A + 1$  must be a multiple of 19, it is easy to check that the only possibility is  $A = 7$ , from which it follows that  $B = 3$ .

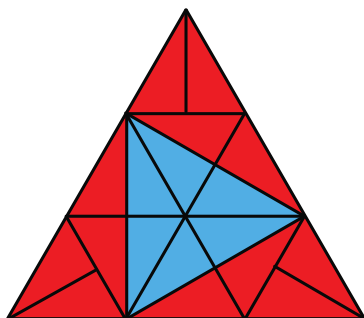
## MEDIUM 2



An equilateral triangle is inscribed inside a larger equilateral triangle, as pictured above. If the big triangle has area 1, what is the area of the smaller equilateral triangle?

Answer:  $1/3$ .

Solution: The big equilateral triangle can be subdivided into 18 equal (congruent) right-angled triangles. The smaller equilateral triangle is filled by 6 of the right-angled triangles, which means it has  $1/3$  the area of the large triangle.



### MEDIUM 3



**Sonya grabs an equal number of marbles in each hand. She transfers 4 of the marbles from her left hand to her right. She counts the remaining marbles in her left hand, throws them away and throws away the same number of marbles from her right hand. Finally, she picks up 5 marbles in her left hand. How many marbles does she have in the end?**

Answer: 13 marbles.

Solution: This is a trick due to Leonardo da Vinci. With Sonya beginning with  $M$  marbles in each hand, the progressive marble totals are as indicated.

	Left	Right
Start:	$M$	$M$
Step 1:	$M - 4$	$M + 4$
Step 2:	0	8
Step 3:	5	8

### MEDIUM 4



**Niels and Évariste each earn \$2000 per month. At the beginning of February Niels receives a pay rise of 10%, and at the beginning of April he receives a pay cut of 10%. Who will earn more money over the year?**

Answer: Niels.

Solution: Évariste will make  $12 \times \$2000 = \$24000$ . Niels makes \$2000 the first month, \$2200 for each of the next two months, and \$1980 for each month after that, for a total of  $\$2000 + 2 \times \$2200 + 9 \times \$1980 = \$24220$ .

## MEDIUM 5



In the picture above does the green rectangle cover more or less than half of the red rectangle?

Answer: More than half.

Solution: The blue triangle pictured below is exactly half of the red rectangle.



## MEDIUM 6



**Carnival Conway offers you a game, where you throw three (normal) dice. If one 6 comes up then you win \$20, for two 6s you win \$50, and if all three dice come up 6 you win \$100; otherwise, you lose \$20. Is this a fair game? How much, on average, would you win or lose?**

Answer: A loss of 69 cents.

Solution: The chances of three 6s is  $1/6 \times 1/6 \times 1/6 = 1/216$ , the chances of exactly two 6s is  $3 \times 1/6 \times 1/6 \times 5/6 = 15/216$ , and the chances of exactly one 6 are  $3 \times 1/6 \times 5/6 \times 5/6 = 75/216$ . That means the chances of no 6s coming up is  $125/216$ , and so the average amount returned on each play of the game is  $100 \times 1/216 + 50 \times 15/216 + 20 \times 75/216 - 20 \times 125/216 = -150/216 = 69$  cents loss.

## MEDIUM 7



**Hermann wants to hike across a desert. The trip will take 6 days but a single person can only carry sufficient water for 4 days. How many helpers will Hermann require to cross the desert? Assume that any helpers are also meant to survive.**

Answer: 2 helpers.

Solution: Clearly 1 helper will not suffice; he could travel with Hermann four 4 days, and give Hermann 2 days of water, but then he'd be stuck. However, if Hermann starts with 2 helpers then Hermann can make it. After 1 day the first helper gives both Hermann and the second helper a day's worth of water, and then heads back. Then, after the second day, the second helper gives Hermann a day's supply. That leaves Hermann with 4 days water, sufficient to cross the rest of the desert, and the second helper has 2 days water to get him back safely.

## MEDIUM 8

$$\frac{1}{A} + \frac{1}{B} = ?$$

**The sum of two numbers is 2 and the product of the same two numbers is 3. What is the sum of the reciprocals of the two numbers?**

Answer:  $2/3$ .

Solution: If  $A$  and  $B$  are the two numbers then  $A + B = 2$  and  $A \times B = 3$ . But then  $1/A + 1/B = (A + B)/(A \times B) = 2/3$ .

## MEDIUM 9



**Poker Poly** deals out cards from a deck one by one. How many cards must **Poly** deal out until he is sure that a flush (five cards of the same suit) has appeared amongst the dealt cards? What if **Poly** wants a straight (five cards in sequence, with an ace counting low or high)?

Answer: 17 and 45.

Solution: With 16 cards **Poly** could have 4 cards in each suit, but then a 17th card ensures at least one suit will have 5 cards. The number of cards required to guarantee a straight is a little trickier. 44 cards may not suffice, because **Poly** may have all the four cards of the following 11 ranks: Ace, 2, 3, 4, 6, 7, 8, 9, Jack, Queen, King. However, if there are 45 cards then there must be cards of at least 12 different ranks, and so there must be a straight amongst them.

## MEDIUM 10



**Something went wrong in a chocolate factory. There are three piles of chocolate bars. Two of the piles consist of bars correctly weighing 100 grams, however all the bars in the third pile weigh 95 grams. Using a digital scale and just one weighing, how do you determine which pile has the dodgy chocolate bars?**

Answer: Take 1, 2 and 3 bars from the piles.

Solution: Take 1 bar from the first pile, 2 bars from the second pile and 3 bars from the third pile. The weight of the 6 bars will then be 595, 590, or 585 grams, depending upon whether the first, second or third pile has the lighter bars.